

CAL vs. CML vs. SML

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Posted in: Level I.

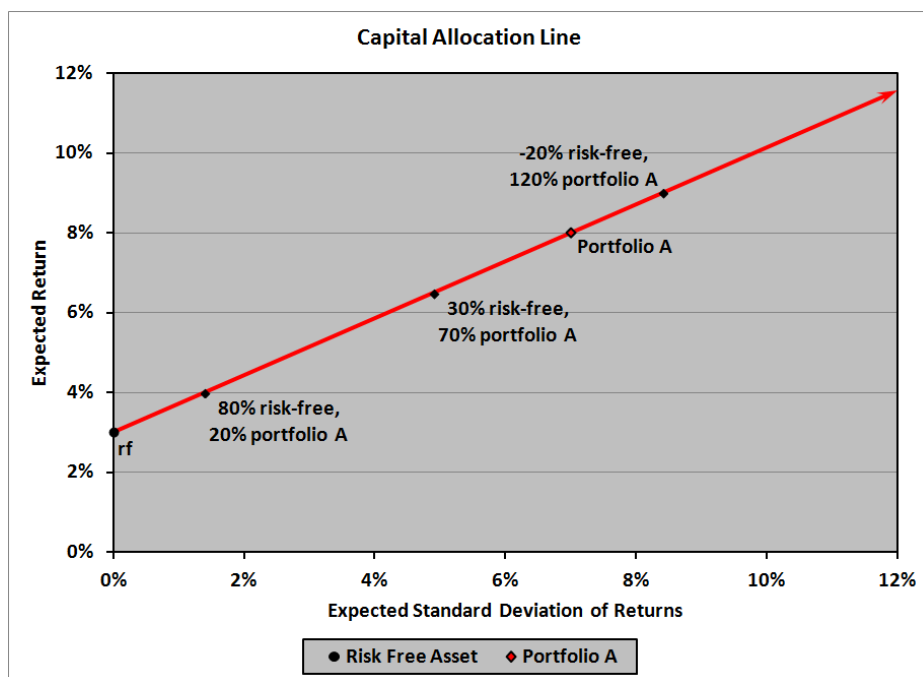
The Capital Allocation Line (CAL), Capital Market Line (CML), and Security Market Line (SML) can be confused easily, and for good reason: the graphs look virtually identical, the assumptions under which they are constructed are essentially the same, and their implications are similar. We'll characterize each one and try to eliminate the confusion.

The assumptions common to these three lines are that:

- There is a risk-free asset (whose standard deviation of returns is zero, and whose expected return is known)
- There are risky assets (whose expected returns, standard deviations of returns, and correlations of returns are known)
- An investor can lend or borrow any amount at the risk-free rate
- An investor can buy any amount of a risky asset

Capital Allocation Line (CAL)

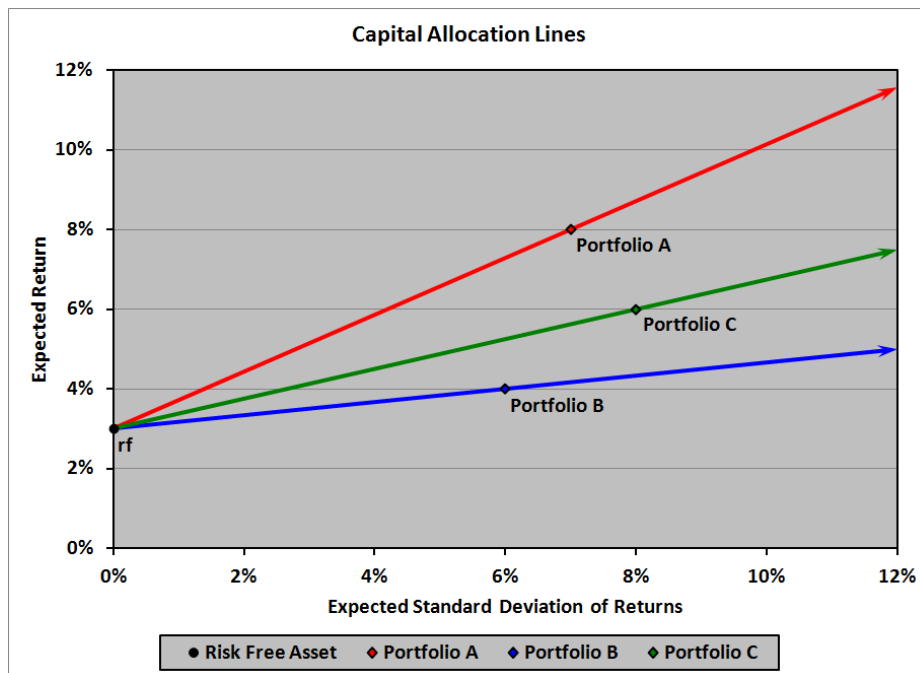
Technically, there isn't a CAL; there are *infinitely many* CALs: one for each risky asset (or portfolio of risky assets). Given a risky asset (portfolio) A and the risk-free asset, all combinations of portfolio A and the risk-free asset lie along the CAL for portfolio A. If, for example, the risk-free asset has an expected return of 3%, and portfolio A has an expected return of 8% with a standard deviation of returns of 7%, then the graph of the CAL for portfolio A is:



Five example portfolios are shown on the line, moving from the lower left to the upper right:

- 100% risk-free asset
- 80% risk-free asset, 20% portfolio A
- 30% risk-free asset, 70% portfolio A
- 100% portfolio A
- -20% risk-free asset (i.e., borrowing 20%), 120% portfolio A

There is a CAL for every risky asset/portfolio. Here is a graph showing CALs for three risky portfolios:



Recall that the slope of a line is the change in y-values between two points divided by the change in x-values between the same two points:

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

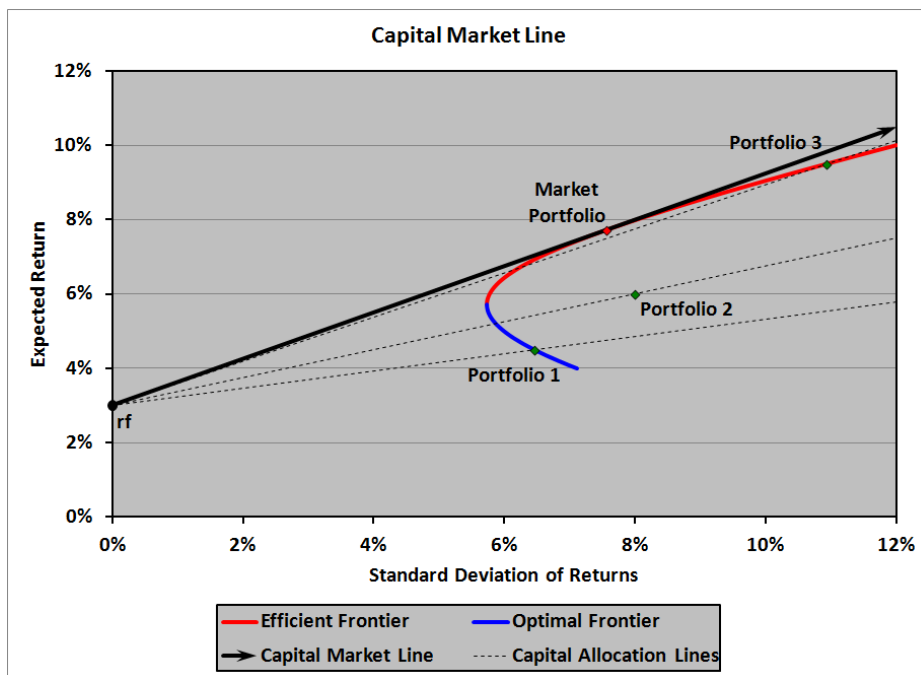
For portfolio A, the two points are (σ_{rf}, r_{rf}) , and $(\sigma_A, E(r_A))$, (where $\sigma_{rf} = 0$), so,

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{E(r_A) - r_{rf}}{\sigma_A - 0} = \frac{E(r_A) - r_{rf}}{\sigma_A}$$

which is the **Sharpe ratio** for portfolio A. Similarly, the slope of the CAL for portfolio B is portfolio B's Sharpe ratio, and the slope of the CAL for portfolio C is its Sharpe ratio.

Capital Market Line

One of the assumptions of Modern Portfolio Theory (MPT) is that all investors have the same market expectations; given that, we can talk about **the** expected return on a portfolio, and **the** Sharpe ratio, and so on. Amongst all of the possible risky portfolios, there is (at least) one that has the highest Sharpe ratio. (Most people say that there is one such portfolio, but it is theoretically possible to have more than one; it's not a big deal either way.) That portfolio will lie on the efficient frontier (see the graph below), and is called the Market Portfolio. (Some people think that this is the portfolio with all investible assets at their total market weights. As far as I can tell, there's no good reason to believe that that's true. It's probably not important.) The CAL for the Market Portfolio is called the Capital Market Line (CML), illustrated here, along with a few other CALs (so you can see that the CML has the highest Sharpe ratio (slope)), the efficient frontier, and the optimal frontier:



Note that the Market Risk Premium (MRP) is defined as:

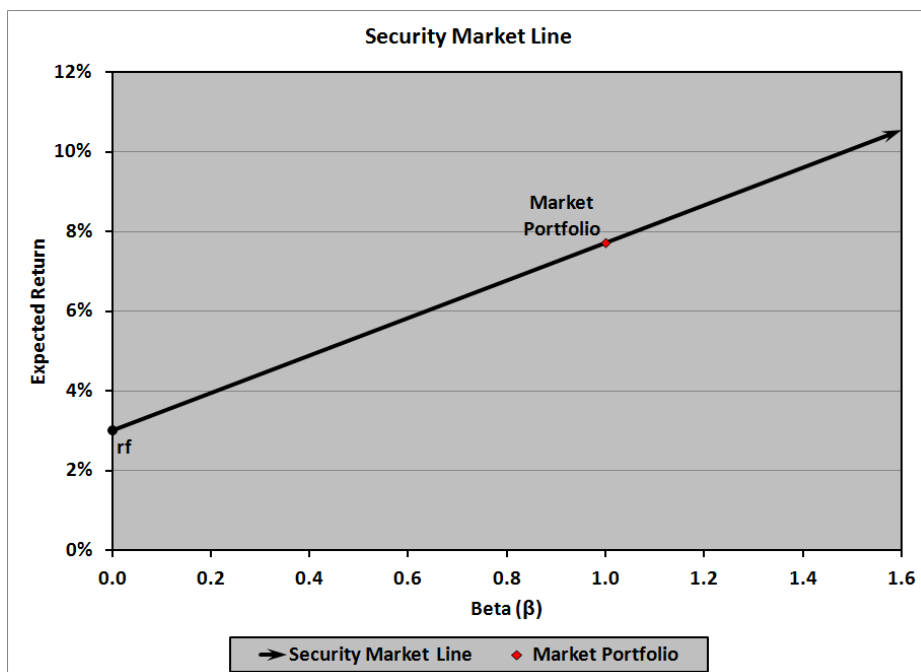
$$MRP = E(r_{mkt}) - r_{rf}$$

so the slope of the CML – the Sharpe ratio for the Market Portfolio – is:

$$slope = \frac{E(r_{mkt}) - r_{rf}}{\sigma_{mkt}} = \frac{MRP}{\sigma_{mkt}}$$

Security Market Line

The security market line (SML) is derived from the CML; the only difference is that the horizontal axis for the SML is systematic risk – beta (β) – whereas the horizontal axis for the CML is total risk – standard deviation (σ) of returns:



The slope of the SML is:

$$\text{slope} = \frac{E(r_{mkt}) - r_{rf}}{1.0 - 0.0} = MRP$$

Implications of the CML and the SML

One of the assumptions of MPT is that securities are fairly priced. If a security is *fairly priced* then it is generating returns that are appropriate for the risk that it poses: it will plot **on** the CML (expected return vs. σ of returns (total risk)), and it will plot **on** the SML (expected return vs. β (systematic risk)).

If a security plots *above* the CML or *above* the SML, then it is generating returns that are too high for the risk it poses: the security is **underpriced**. (For example, suppose that a security is priced at \$10 and is earning a 10% return (\$1.00) when, according to its risk it should be earning a 5% return. A return of \$1.00 is 5% of \$20, so the security should be priced at \$20: it is underpriced.)

If a security plots *below* the CML or *below* the SML, then it is generating returns that are too low for the risk it poses: the security is **overpriced**.